Ready

Grades: 6-8

- Teacher Toolkit Trial
- Lesson Sample
- Student Discourse Cards
Lesson 2
Understand Subtraction of Positive and Negative Integers

MGSE Focus

Domain
The Number System

Cluster
Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

Standard
MGSE7.NS.1c

Standards for Mathematical Practice (SMP)
4 Model with mathematics.
8 Look for and express regularity in repeated reasoning.

Lesson Objectives

Content Objectives
• Subtract rational numbers by adding the additive inverse.
• Use subtraction and absolute value to find the distance between two numbers on a number line.
• Find the distance between two points on a coordinate plane that have either the same x- or y-value.

Prerequisite Skills
• Understand inverse relationships.
• Add integers.
• Find absolute values.

Lesson Vocabulary
There is no new vocabulary.

Learning Progression

In Grade 6 students learned about negative numbers, opposites, and absolute value. They learned to locate negative numbers on a number line. Earlier in Grade 7 students have learned about adding signed numbers. In this lesson students learn to subtract negative numbers by adding the opposite. They also learn to find the distance between two points by finding the absolute value of the difference of their coordinates.

In Grade 8 as students continue their study of mathematics, operations with integers will be a basic tool in algebra. They will need to understand the concepts behind the operations as well as perform them with precision and fluency.
## Lesson Pacing Guide

### Whole Class Instruction

<table>
<thead>
<tr>
<th>Day</th>
<th>45–60 minutes</th>
<th>Toolbox: Interactive Tutorial</th>
</tr>
</thead>
<tbody>
<tr>
<td>Day 1</td>
<td>45–60 minutes</td>
<td>Understanding Adding and Subtracting Positive and Negative Numbers</td>
</tr>
</tbody>
</table>

**Introduction**
- Think It Through Question 10 min
- Think 15 min
- Think 15 min
- Reflect 5 min

**Practice and Problem Solving**
Assign pages 11–12.

<table>
<thead>
<tr>
<th>Day 2</th>
<th>45–60 minutes</th>
<th>Guided instruction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Think About Subtracting Positive and Negative Integers</td>
</tr>
</tbody>
</table>

- Let's Explore the Idea 15 min
- Let's Talk About It 20 min
- Try It Another Way 10 min

**Practice and Problem Solving**
Assign pages 13–14.

<table>
<thead>
<tr>
<th>Day 3</th>
<th>45–60 minutes</th>
<th>Guided Practice</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Connect Ideas About Subtracting Positive and Negative Integers</td>
</tr>
</tbody>
</table>

- Compare 10 min
- Explain 10 min
- Analyze 5 min

**Practice and Problem Solving**
Assign pages 15–16.

### Small Group Differentiation

#### Teacher-Toolbox.com

**Reteach**

**Ready Prerequisite Lessons** 45–90 min

**Grade 6**
- Lesson 12 Understand Positive and Negative Numbers
- Lesson 13 Absolute Value and Ordering Numbers

**Grade 7**
- Lesson 1 Understand Addition of Positive and Negative Integers

#### Teacher-led Activities

**Tools for Instruction** 15–20 min

**Grade 7**
- Subtract Positive and Negative Rational Numbers

### Personalized Learning

#### i-Ready.com

**Independent**

**i-Ready Lessons** 15–20 min

**Grade 7**
- Understanding Adding and Subtracting Positive and Negative Numbers
Lesson 2 Understand Subtraction of Positive and Negative Integers

At A Glance

Students explore the subtraction of signed numbers by comparing it with the addition of positive and negative numbers. They also explore how to rewrite subtraction problems as addition problems.

Step By Step

Think It Through

- Introduce the Question at the top of the page.

▶ English Language Learners

- Review the process of using a number line to add a positive and a negative number.
- Talk about the inverse relationship between addition and subtraction and how to use it to write related number sentences.

▶ Mathematical Discourse 1

- Show how to use a number line to model subtraction.
- Connect the representations to show that a subtraction problem can be written as an addition problem by adding the opposite of the second number.

SMP TIP Model with Mathematics

Using a number line to model problems with mathematics helps students grasp the reasons behind the rules. Throughout the lesson, help students see the connections among the number lines, numerical expressions, and situations described in the problems. (SMP 4)

▶ Mathematical Discourse 2

1 What does it mean when we say that addition and subtraction are inverse relationships? Students could say that addition puts together and subtraction takes apart.

2 When you add two numbers, is the sum always greater than the addends? Support your answer with examples using different types of numbers. Students should note that with counting numbers, the sum is always greater; with 0, the sum is the same as the other addend. When the addends are both negative, the sum is less than either addend. Discuss with students the situations that can occur when one addend is positive and the other addend is negative.

▶ English Language Learners

- Display 3 + (−2). Read, 3 plus negative 2. Say: This expression shows adding a negative number.
- Now display −5 − 4. Read, negative 5 minus 4. Say: This expression shows subtracting a positive number.
- Continue with other expressions, such as −5 + (−3) and 6 − 2. Call on students to read and describe each expression.
Think: How do you write a subtraction problem as an addition problem?

Look at the two equivalent expressions on the previous page.

\[ 5 - 3 \]
\[ 5 + (-3) \]

What operation is the opposite of subtraction? What is the opposite of a number? 

The first number stays the same. Use the opposite operation. Use the opposite of the second number.

This means that every subtraction problem can be written as an addition problem.

So, if you know how to add positive and negative numbers, you know how to subtract them. Here are some other examples:

\[ 4 - (-4) = 0 \]
\[ 4 + (-4) = 0 \]

\[ 1 - 3 = -2 \]
\[ 1 + (-3) = -2 \]

Reflect

1. Why can you write a subtraction problem as an addition problem? How do you write a subtraction problem as an addition problem?

You can write a subtraction problem as an addition problem because subtraction is related to addition. They are opposite operations. To write a subtraction problem as an addition problem, you add the opposite of the second number to the first number.

Concept Extension

Use a pattern to subtract integers.

- Draw a table similar to this on the board.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>-1</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>-2</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>-3</td>
</tr>
</tbody>
</table>

- Point out that the numbers in the second column decrease by one.
- Direct students to complete the next three rows through \( 3 - 0 \) using subtraction.

- Examine the third column. Once students notice that the differences increase by 1, have them apply the pattern to complete the table.
- Direct students to apply the rule of adding opposites to show that both the pattern and the rule work.

Mathematical Discourse

3. How would you rewrite \( 8 - (-2) \) as addition? How would it work?

Students should say that the expression \( 8 - (-2) \) is equivalent to \( 8 + 2 \) because you add the opposite of \(-2\).

4. How could you explain subtracting a negative number to a friend?

Students could include the idea that you add the opposite, which is a positive number.
Lesson 2: Understand Subtraction of Positive and Negative Integers

Guided Instruction

At A Glance

Students rewrite subtraction problems as addition problems. They model the answer using number lines. Students also connect the process of finding the distance between two numbers on a number line with finding the absolute value of the difference of the numbers.

Step By Step

Let's Explore the Idea

- Tell students that they will have time to work individually on the problems on this page and then share their responses in groups. You may choose to work through the first problem together as a class.
- As students work individually, circulate among them. This is an opportunity to assess student understanding and address student misconceptions. Use the Mathematical Discourse questions to engage student thinking.

Mathematical Discourse 1 and 2

- Encourage students to make sure their subtraction problems model the information in the word problems before they rewrite each as an addition problem.
- Take note of students who are still having difficulty, and wait to see if their understanding progresses as they work in their groups during the next part of the lesson.

Student Misconception Alert

When subtracting a negative number, students sometimes change the subtraction sign to an addition sign without changing the negative number to a positive. Go over the rule again, explaining that they must change both the subtraction sign and the negative sign.

Mathematical Discourse

1. Explain why your answer to problem 3 makes sense.
   Students may say that Renee needs 5 points to get up to 0 and then 25 more to get up to 25. Altogether she needs 30 points.

2. What is another way to solve problem 4 without using the equation $30 - (-10) = 40$?
   Students might say that Rob usually reads 30 minutes, but one Saturday he was short by 10 minutes. He has to add that 10 minutes to the 30 minutes he will read this Saturday. The equation for this situation as phrased is $30 + 10 = 40$. 

Let's Explore the Idea

Think About

Subtracting Positive and Negative Integers

You can write a subtraction problem as an addition problem.

In problems 2-5, write a subtraction problem to represent the situation. Then write the subtraction problem as an addition problem. Model the addition problem on a number line, and use the number line to answer the question.

2. Adam buys 9 gift cards and gives 6 away. How many does he have left?
   $9 - 6; 9 + (-6); Adam has 3 cards left.$

3. Renee is playing a game online. If she gets a total of 25 points, she will have a new high score. She currently has $-5$ points. What is the difference between a high score of 25 points and the number of points she currently has?
   $25 - (-5); 25 + 5; The difference is 30 points.$

4. Rob is trying to read for 30 minutes each Saturday. He only read for 20 minutes last Saturday. He represents the amount of time he was short of the total 30 minutes as $-10$ minutes. This Saturday, he wants to make up the difference between the number of minutes he usually reads on Saturday and the number of minutes he was short last Saturday. How many minutes will Rob need to read this Saturday?
   $30 - (-10); 30 + 10; Rob will need to read for 40 minutes.$

5. The temperature at noon is $-4^\circ F$. The temperature at 6:00 AM is $-12^\circ F$. What is the difference between the noon and the 6:00 AM temperatures?
   $-4 - (-12); -4 + 12; The difference in the temperatures is $8^\circ F.$

Vi: Used with tw

Disp: 6 t col
Ask: Ask:
Have been equal
Say from to fill two t
Disp expla to 4 -
Repe
Let's Talk About It

Solve the problems below as a group.

6. Fran rewrites a subtraction problem as an addition problem. The addition problem she writes is \(-3 + (-4)\). How could you use a number line to help you solve \(-3 + (-4)\) as a subtraction problem? Possible answer: I could represent \(-3 + (-4)\) on a number line and then think of a subtraction problem that could be modeled by that same situation.

7. Look back at problem 5. What was Fran's original subtraction problem? Explain how you got your answer. The original subtraction problem was \(-3 - 4\). Possible answer: To get this answer, leave \(-3\) alone; take the opposite of addition, which is subtraction; then write the opposite of \(-4\), which is 4.

Use the number line below for problems 8-10.

8. What is the distance between 2 and 3 on the number line? 3 unit
   What is \(|-3 - 2|?\) 1 unit
   What is \(|-3 + 2|?\) 1 unit

9. What is the distance between 4 and 1 on the number line? 3 units
   What is \(|-4 - 1|?\) 5 units
   What is \(|-4 + 1|\) 3 units

10. Look at your answers to problems 8 and 9. What do you notice about the absolute value of the difference between two numbers? The absolute value of the difference is equal to the distance between the two points.

Try It Another Way

Work with your group to solve this problem.

11. Write an absolute value expression to represent the distance between \(-2\) and 4 on a number line. Then evaluate the expression.
    \(|-2 - 4| = |-6| = 6\)

Visual Model

Use a number line to model another way to find the distance between two integers.

- Display a number line that extends from \(-6\) to 6. Circle the 4 and the \(-2\) with colored chalk.
- Ask: How many units is 4 from 0? [4].
- Ask: How many units is \(-2\) from 0? [2]
- Have students add to find the distance between the two points. Display the equation 4 + 2 = 6.
- Say that subtracting the lesser number from the greater number is another way to find the distance between two numbers.
- Display 4 - (-2) = 6. Have students explain why this equation is equivalent to 4 + 2 = 6.
- Repeat with other integers.

Mathematical Discourse

3. **Can a distance ever be negative? Explain.**
   A negative sign indicates direction, not distance. Students should discuss that a distance must be nonnegative because it tells only how far apart two things are, not which direction one is from the other.

4. **Why is absolute value an important tool for subtracting to find distances?**
   Listen for the idea that using absolute value ensures that the distance is always nonnegative.

Mathematical Discourse 3 and 4

- When students use number lines to find the distance between two integers, encourage them to relate the visual solution to the idea that subtracting is the same as adding the opposite.

Visual Model

Try It Another Way

- Direct the groups' attention to Try It Another Way. Have a volunteer from each group come to the board to draw and explain the group's solution to problem 11.

Assign Practice and Problem Solving pages 13-14 after students have completed this section.
**Lesson 2 Understand Subtraction of Positive and Negative Integers**

**At A Glance**
Students demonstrate their understanding of subtracting negative numbers. They use subtraction to find distances on a coordinate plane.

**Step By Step**
- Discuss each **Connect** problem as a class using the discussion points outlined below.

**Compare**
- Direct pairs of students to compare the expressions $8 - 15$ and $8 + (-15)$.
- Ask: *Why do the two expressions simplify to the same number?* In $8 - 15$, the number being subtracted is $-15$. In $8 + (-15)$, the number being added is $-15$. Subtracting a number produces the same result as adding the opposite of that number.

**SMP TIP Repeated Reasoning**
Students look for and express regularity in repeated reasoning when they solve subtraction problems by using related addition problems. Encourage them to keep in mind the inverse relationship between addition and subtraction as they answer the questions on this page. (*SMP 8*)

**Explain**
- This problem requires students to explain the concept and algorithm clearly.
- Have volunteers explain why and how it is possible to change a subtraction problem into an addition problem. (You can write a subtraction problem as an addition problem because the operations are inverses of each other.)

**Analyze**
- Encourage students to try a few concrete examples before they attempt to give the answer.
- Ask: *Why is it necessary to use absolute value for the distance when the greater number is subtracted from the lesser number?* (because distance must always be expressed as a nonnegative number)

**Scoring Rubrics**
See student facsimile page for possible student answers.

### Part A

<table>
<thead>
<tr>
<th>Points</th>
<th>Expectations</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Students found the correct answers and provided both an addition and a subtraction expression for each.</td>
</tr>
<tr>
<td>1</td>
<td>Students either found the correct answers but did not provide the expressions or provided correct expressions but not the correct answers.</td>
</tr>
<tr>
<td>0</td>
<td>The answers were incorrect and the expressions were either incorrect or not shown.</td>
</tr>
</tbody>
</table>
**Subtracting Positive and Negative Integers**

**Apply**

**Put It Together** Use what you have learned to answer the questions below.

The map of Jean's neighborhood shows the location of Jean's house, her school, her friend Pam's house, and her favorite restaurant.

![Map Diagram]

**Part A**

Find each distance described below by finding the absolute value of the difference between the x-coordinates of the two points on the map. Write a subtraction problem and a related addition problem for each distance. Then evaluate your expressions to find the distance.

<table>
<thead>
<tr>
<th>Restaurant to Pam's House</th>
<th>Pam's House to Jean's House</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subtraction problem</td>
<td>Addition problem</td>
</tr>
<tr>
<td>[4 - (-3)]</td>
<td>[4 + 1]</td>
</tr>
<tr>
<td>Distance 5 units</td>
<td>Distance 2 units</td>
</tr>
</tbody>
</table>

**Part B**

Refer to the map above. What coordinates do you subtract to find the distance from Jean's house to her school? Explain your reasoning. **You need to subtract the y-coordinates.** Possible explanation: The x-coordinates are the same, so if you subtracted them, you would get a distance of 0.

**Part C**

Write a subtraction problem and a related addition problem for the distance described below. Then evaluate your expressions to find the distance.

<table>
<thead>
<tr>
<th>Jean's house to school</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subtraction problem</td>
</tr>
<tr>
<td>[2 - (4)]</td>
</tr>
<tr>
<td>Addition problem</td>
</tr>
<tr>
<td>[2 + (-4)]</td>
</tr>
<tr>
<td>Distance 6 units</td>
</tr>
</tbody>
</table>

**Step By Step**

- Direct students to complete the Put It Together task on their own.
- Point out that the answer must show both a subtraction expression and its related addition expression.
- As students work on their own, walk around to assess their progress, answer questions, and give additional support, if needed.
- If time permits, have students share the reasoning they used to get the answers.

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PRACTICE AND PROBLEM SOLVING

Assign Practice and Problem Solving pages 15-16 after students have completed this section.

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**Points | Expectations**

| 2       | Students said that they used the y-coordinates and explained their reasoning. |
| 1       | Students said that they used the y-coordinates but did not explain why.       |
| 0       | Students answered incorrectly.                                               |

**Part C**

<table>
<thead>
<tr>
<th>Points</th>
<th>Expectations</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Students found both expressions and the correct distance.</td>
</tr>
<tr>
<td>1</td>
<td>Students found one correct expression and the correct distance.</td>
</tr>
<tr>
<td>0</td>
<td>Students found no expressions or distance.</td>
</tr>
</tbody>
</table>
Lesson 2
Understand Subtraction of Positive and Negative Integers

Differentiated Instruction

▶ Intervention Activity
Find distances between points on a number line.

- Prepare slips of paper labeled with integers from 
  −10 through 10, enough for all the students in
  the class.
- Draw a large number line from −10 through 10 all the
  way across the board. Leave enough space for students
  to stand in front of the numbers.
- Give each student a slip of paper. Call two students to
  the board to stand in front of the numbers, corresponding
  to those on their papers.
- Have a volunteer come to the board and write an
  expression that can be simplified to tell the distance
  between the integers.
- Have another volunteer explain how to simplify
  the expression.
- Finally, have the class verify the answer by visually
  finding the distance between the two students.

▶ On-Level Activity
Use thermometers to subtract integers.

- Have each student draw a thermometer with a scale of
  1 from 10°C to −10°C.
- Say to students: Suppose the temperature outside is 8°C
  and drops by 5°C. What is the new temperature? [3°C]
- Have students find the new temperature mentally, and
  then confirm it using the thermometer. Display
  8 − 5 = 3°C.
- Now say: Suppose the temperature is 3°C and drops
  by 4°C. What is the new temperature? [−1°C]
- Again, have students find the new temperature
  mentally, and then confirm it using the thermometer.
  Display 3 − 4 = −1°C.
- Finally, say: Suppose the temperature is −1°C and drops
  by 5°C. What is the new temperature? [−6°C] Display
  −1 − 5 = −6°C.
- Have students study the displayed equations. Discuss
  how students might use addition instead of
  subtraction to produce the same results.

▶ Challenge Activity
Connect subtraction problems to real-world contexts.

- Display several expressions involving the subtraction of integers. Include several examples such
  as −3 2 6, 4 2 (−4), and −6 2 (−1).
- Have students choose two expressions, including one that involves subtracting a negative integer.
- Instruct students to write a word problem that can be modeled by their chosen expressions.
  Have them solve the problems and make sure the answers make sense.
- Have each student share one of their word problems with the class.
Overview
Assign the Lesson 2 Quiz and have students work independently to complete it.

Use the results of the quiz to assess students' understanding of the content of the lesson and to identify areas for reteaching. See the Lesson Pacing Guide at the beginning of the lesson for suggested instructional resources.

Tested Skills
Assesses MGSE7.NS.1c
Problems on this assessment form require students to be able to subtract rational numbers by adding the additive inverse, use subtraction and absolute value to find the distance between two numbers, and find the distance between two points on the coordinate plane that have either the same x- or y-value. Students will also need to be familiar with understanding inverse relationships, adding integers, and finding absolute value.

Lesson 2 Quiz continued

Alana makes and sells homemade bracelets. She made a profit of $65 last week. This week, she donated some bracelets to charity. Her profit at the end of this week can be represented by $-25.$

Part A
Write a subtraction expression and an addition expression to represent Alana's total profit for the two weeks combined.

Subtraction expression: __________
Addition expression: __________

Part B
What is the difference between last week's profit and this week's profit?
Show your work.

Answer: __________

Laura says that the distance between (4, -2) and (4, -7) in the coordinate plane is equal to $|2 - (-7)|$ units.
Ryan says that the distance between the points (2, 3) and (-9, 3) in the coordinate plane is equal to $|2 + 9|$ units.
Is either Laura or Ryan correct? Explain.

______________________________
Common Misconceptions and Errors

Errors may result if students:
- overlook negative signs when interpreting integers.
- forget that positive and negative numbers are on the opposite side of 0 on a number line.
- fail to change a negative number to positive when changing the subtraction of a negative number to an addition problem.
- ignore absolute value symbols when evaluating expressions.

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Lesson 2 Quiz Answer Key

1. A, D
   DOK 1

2. a. True
   b. False
   c. True
   d. False
   DOK 2

3. B, F
   DOK 2

4. Part A:
   65 - 25
   65 + (-25)
   DOK 2
   Part B:
   90
   DOK 2

5. Both Laura and Ryan are correct. Possible explanation: The distance between two points in a coordinate plane that lie on a horizontal or vertical line is the absolute value of the difference of the coordinates that are not equal. So, the distance between (4, –2) and (4, –7) is equal to |–2 - (–7)|, or 5 units. The distance between (2, 3) and (–9, 3) is equal to |2 - (–9)|, which is equivalent to |2 + 9|, or 11 units.
   DOK 3