

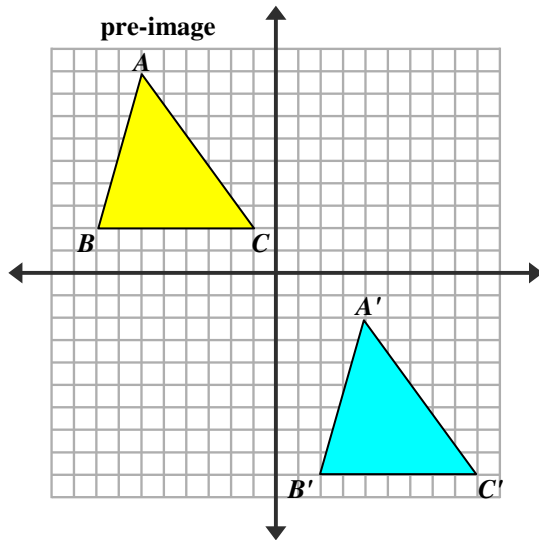
## TRANSFORMATIONS

Students should be able to apply transformations to different objects. They should describe transformations as functions that take points in the plane as inputs and give other points as outputs – making connections to functions and the transformations they have done on functions. Students should also make comparisons between transformations that preserve distance and angle to those that do not. They should be able to predict whether or not a transformation will maintain the congruence between the pre-image and image – i.e. the input object is congruent to the output object.

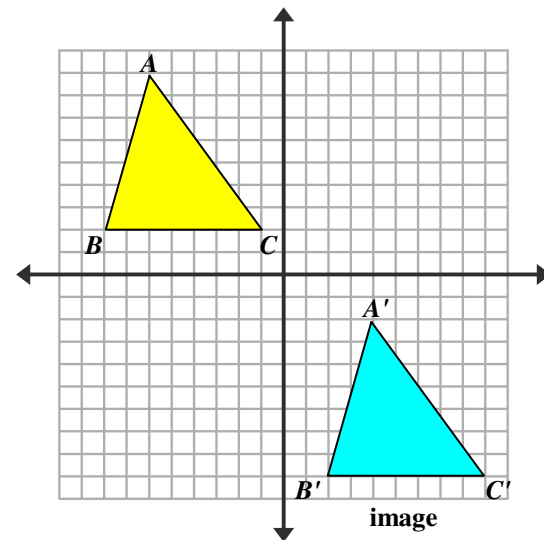
**NOTE:** Using software that allows students to manipulate objects and perform these transformations is highly recommended. For example: Geometer's Sketchpad, Geogebra and other programs. These often come with materials, diagrams that the software company has already created to facilitate explorations of these topics.

### Transformation Terminology

**pre-image:** original figure before transformation; input into the transformation function



**image:** figure after transformation; output from the transformation function

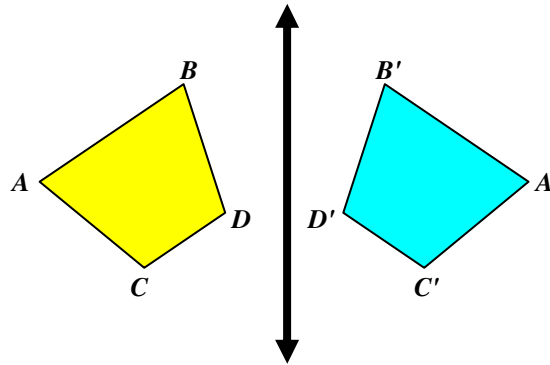


## TRANSFORMATIONS (rigid motion transformations)

**Rigid Motion Transformations:** preserve congruence

- **reflection** – flip

- The reflection of an object is called its *image*. If the original object (which is called the *pre-image*) was labeled with letters, such as polygon  $ABCD$ , the image may be labeled with the same letters followed by a *prime* symbol,  $A'B'C'D'$ .

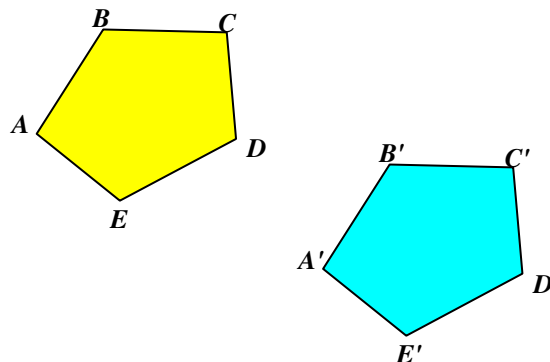


- The line (where a mirror may be placed) is called the *line of reflection*. The distance from a point to the line of reflection is the same as the distance from the point's image to the line of reflection.
- A reflection can be thought of as folding and "flipping" an object over the line of reflection.
- Connecting this to functions – the pre-image would be the input to the transformational function and the image would be the output.
- Connect this to the idea of absolute value and distance from some origin.

- **translation** – slide

- A *translation* "slides" an object a fixed distance in a specified direction. The original object and its translation have the **same shape and size**, and they **face in the same direction**, they are oriented the same way. The word "translate" in Latin means "carried across".
- If the pre-image (input was polygon  $ABCDE$ , then the image after a translation has taken place would be polygon  $A'B'C'D'E'$ .
- A translation is strictly additive—only adding values, as opposed to dilating (scale or multiplication) an object—this would not preserve congruence.

Think of polygon  $ABCDE$  as sliding two units to the right and one unit down. Its new position is labeled  $A'B'C'D'E'$ .

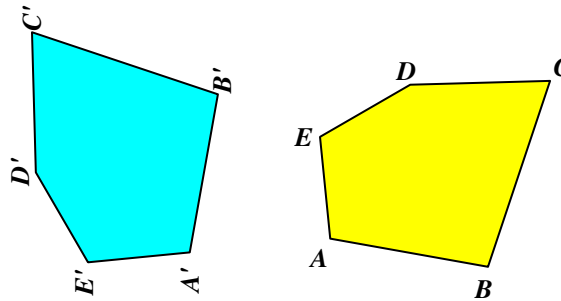


Think of polygon  $ABCDE$  as sliding two units to the right and one unit down. Its new position is labeled  $A'B'C'D'E'$ .

## TRANSFORMATIONS (rigid motion transformations - continued)

**Rigid Motion Transformations:** preserve congruence

- **rotation** – turn
  - A rotation turns an object around a point. Rotations can occur in either a **clockwise** (to the right) or **counterclockwise** (to the left) direction.



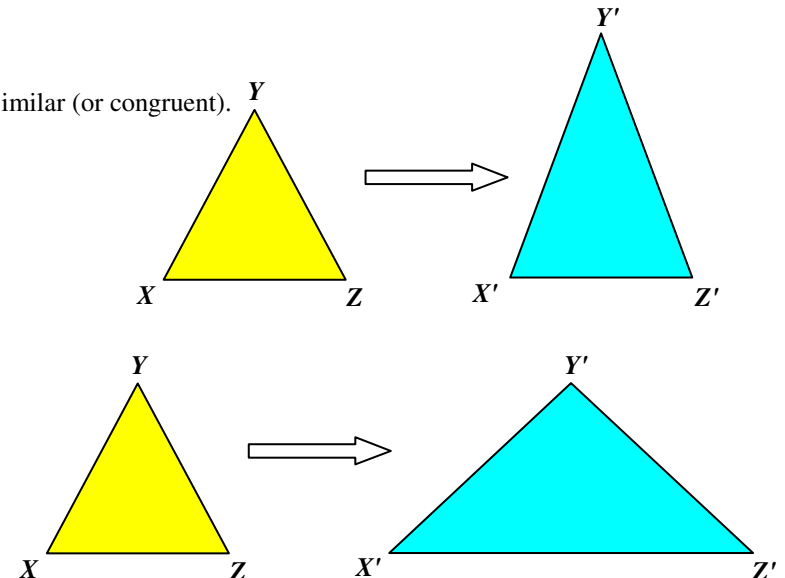
This rotation of the pre-image polygon  $ABCDE$  is  $90^\circ$  counterclockwise. The output is the image  $A'B'C'D'E'$ .

- A **positive angle** of rotation turns the figure **counterclockwise**, and a **negative angle** of rotation turns the figure in a **clockwise direction**.
- Notice that a rotation does not change the size of the figure.

## TRANSFORMATIONS (non-rigid motion transformations)

**Non-Rigid Transformations:** do not preserve congruence

- **dilation** – A dilation is a multiplicative stretch which produces a similar figure. The stretch is the same both horizontally and vertically. The amount by which a figure grows or shrinks is called the scale factor.
- **horizontal stretch** – a **stretch** in which a **plane figure** is distorted **horizontally**.
  - The size and shape of the figure is changed so the image and pre-image are no longer similar (or congruent).
- **vertical stretch** – a **stretch** in which a **plane figure** is distorted **vertically**.
  - The size and shape of the figure is changed so the image and pre-image are no longer similar (or congruent).



**EXAMPLE (translation in the coordinate plane)**

**NOTE:** reflection in the origin is the same thing as a 180° rotation about the origin.

<i>Reflections in the Coordinate Plane</i>				
Reflection	<i>x</i> -axis	<i>y</i> -axis	origin	<i>y = x</i>
<b>Pre-image to Image</b>	$(a, b) \rightarrow (a, -b)$	$(a, b) \rightarrow (-a, b)$	$(a, b) \rightarrow (-a, -b)$	$(a, b) \rightarrow (b, a)$
<b>How to find coordinates</b>	Multiply the <i>y</i> -coordinate by -1.	Multiply the <i>x</i> -coordinate by -1.	Multiply both coordinates by -1.	Interchange the <i>x</i> - and <i>y</i> -coordinates.
<b>Example</b>				