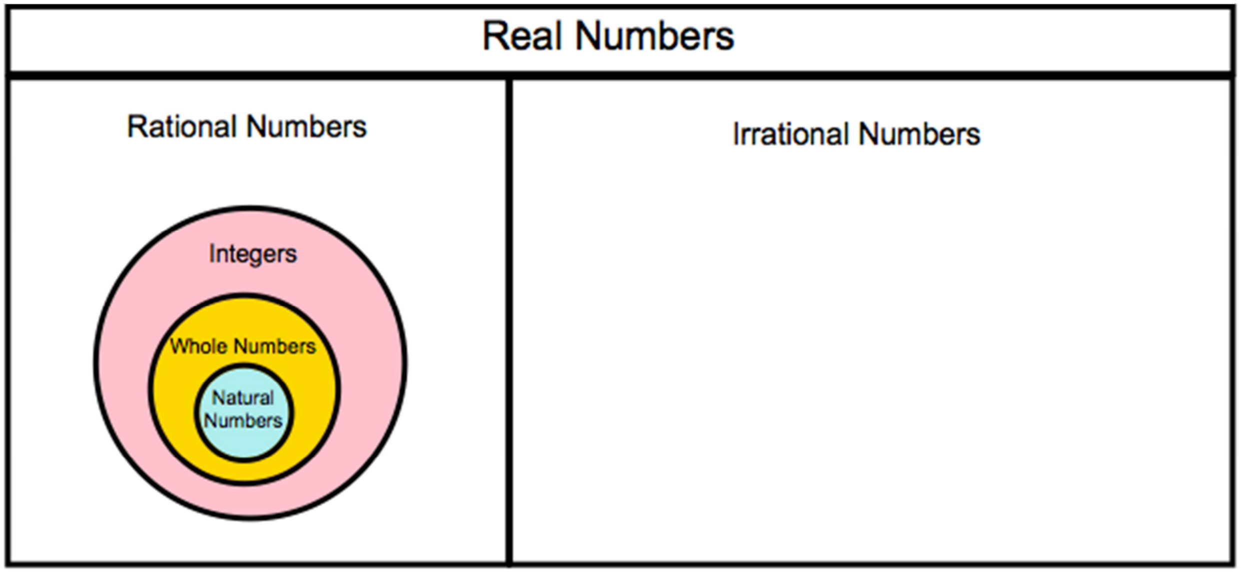
What is the difference between a rational and an irrational number?

- A rational number can be expressed as a fraction. Any terminating ($.25$) or repeating decimal number ($1.\overline{3}$) can be expressed as a fraction (see below—convert repeating decimal to a fraction).
- An irrational number has a non-repeating decimal for which we can only estimate the fraction (see estimation process below).



The Venn diagram is an illustration of the Real Number system. Some of the things that might be discussed with students:

- Real Numbers are a class of numbers. There are other numbers that students will be studying later in secondary school (e.g. Complex Numbers).
- Although the diagram has "boxes" for Rational v. Irrational numbers, the set is actually infinite. The set of Intgers, Whole Numbers, and Natural numbers are also infinite.
- The notion of "countability" is an interesting and accessible for students in 8th grade. The set of Rational numbers is countable, while the set of Irrational numbers is not. See http://www.learner.org/courses/mathilluminated/units/3/textbook/05.php students often think that there are "fewer" Irrational Numbers than Rational.
- Geometric representations of irrational numbers are often more intuitive for students see: http://www.hyperflight.com/golden_numbers-proportion.htm or http://www.principlesofnature.net/conclusions_for_the_arts_and_sciences/towards_new_geometric_number_notations.htm

Converting a repeating decimal into a fraction

Students are often taught that to convert a repeating decimal to a fraction, simply write the repeating element over the necessary number of 9's i.e. .27272727272727… will be 29/99 or .123123123123123…is 123/999, but students do not understand the algorithm. Below is a brief explanation:

Example:

Write $0.\overline{48}$ as a fraction.

Because $0.\overline{48}$ does not terminate, we cannot say that it is an exact portion of a power of 10. The problem is the repeating 48. We need to somehow get rid of the repeating decimal. We start by setting x= $0.\overline{48}$. What we are saying is that x=.48484848… Now, let's multiply each side of the equation x=.48484848…by 100. We get:

100x=48.4848484848…

This came from the original equation x=.48484848… thus we have two equations:

100x=48.48484848…
x=.48484848…

If we subtract the second equation from the first, we get: 99x=48

No repeating decimal. We can now solve for x: X=48/99

Approximating square roots without a calculator

Example: find $\sqrt{6}$ to (up to 4 decimal places)

- Since $2^2 = 4$ and $3^2 = 9$, we know that √6 is between 2 and 3.
- Let's just make a guess of it being 2.5.
- Squaring that we get $2.5^2 = 6.25$—too high, make a slightly smaller guess
- Let's try 2.4 next.
- To find approximation to four decimal places we need to do this till we have five decimal places, and then round the result.

Guess	Square of guess	High or low
2.4	5.76	Too low
2.45	6.0025	Too high, close
2.449	5.997601	Too low
2.4495	6.00005025	Too high, so between 2.2449-2.4495
2.4493	5.99907049	Too low
2.4494	5.99956036	Too low, so between 2.4494-2.4495
2.44945	5.9998053025	Too low, so between 2.44945-2.4495
Rounded to 2.4495		